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## Summary

- We propose algorithm to train VAE model with data-driven prior
- We propose simple and efficient algorithm for incremental learning which shares prior knowledge between tasks, keeping the single encoder-decoder pair.
- We empirically validate the proposed algorithm on commonly used benchmark datasets (MNIST, and Fashion-MNIST) for both offline and incremental setting.

## Objectives

- Use data-driven prior to train VAE
- Construct feasible approximation for the optimal prior, avoiding overfitting
- Reduce catastrophic forgetting in incremental learning setting, using data-driven prior

## Optimal Prior

$$\log p(x) \geq \mathcal{L}(x; \theta; q) = \mathbb{E}_{z \sim q(z)} [\log p_\theta(x|z)] - D_{\text{KL}}[q(z) \| p(z)],$$

Optimal prior in terms of Empirical Bayes:

$$p^*(z) = \arg \max_{p(z)} \mathcal{L} = \frac{1}{N} \sum_{n=1}^N q_\phi(z|x_n).$$

## Boosting for density estimation

Approximates complex distribution by the simple mixture

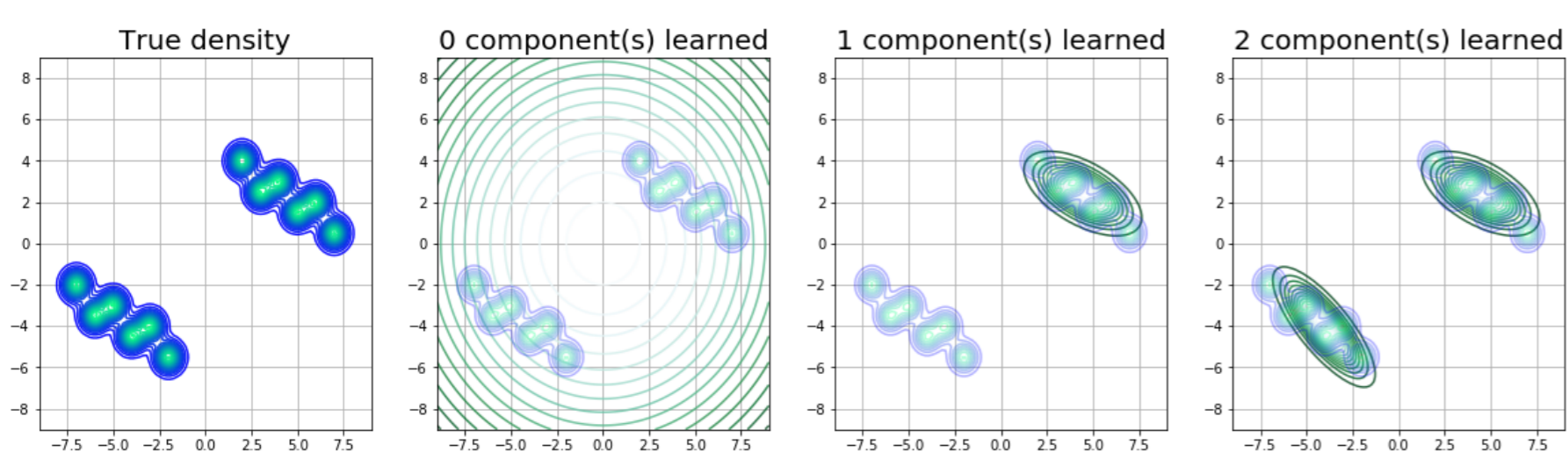
$$p^* \approx \sum_{i=1}^K \alpha_i p^{(i)} = p_K$$

New component  $h$  is learned greedily, using MaxEntropy approach

$$\max_{h \in Q} \mathcal{H}(h)$$

$$D_{\text{KL}}(p_{t-1}|p^*) - D_{\text{KL}}(p_t|p^*) > 0$$

+ linearization



## BooVAE

**Input:** Dataset :  $\{(x_i)\}_{i=1}^N$

**Input:**  $\lambda$ , Maximal number of components  $K$

Choose random subset  $\mathcal{M} \subset \mathcal{D}$

Initialize prior  $p_0 = \mathcal{N}(\mu_0, \Sigma_0)$

$\theta^*, \phi^*, \mu_0, \Sigma_0 = \mathcal{L}(p_0, \theta, \phi)$

$k = 1$

**while** not converged **do**

Update network parameters  $\theta^*, \phi^* = \arg \max \mathcal{L}(p_{k-1}, \theta, \phi)$

**if**  $k < K$  **then**

Update optimal prior  $p^*(z) = \frac{1}{n} \sum_{x \in \mathcal{M}} q_{\phi^*}(z|x)$

Add new component  $p_k = \alpha h + (1 - \alpha)p_{k-1}$

$$h = \arg \min D_{\text{KL}}(h \| \left[ \frac{p^*}{p_{k-1}} \right]^\lambda)$$

$$\alpha = \arg \min D_{\text{KL}}(\alpha h + (1 - \alpha)p_{k-1} \| p^*)$$

$k = k + 1$

**end if**

**end while**

**return**  $p_K, \theta^*, \phi^*$

## Results

# comp.	MNIST		Fashion MNIST	
	Vamp	Boo	Vamp	Boo
10	90.39	<b>89.98</b>	232.53	<b>231.94</b>
20	89.97	<b>89.78</b>	232.22	<b>231.84</b>
50	89.40	<b>89.16</b>	232.19	<b>231.63</b>
100	89.16	<b>88.90</b>	232.01	<b>231.55</b>
500	88.82	<b>88.68</b>	<b>231.67</b>	231.85

Table: NLL, Offline setting

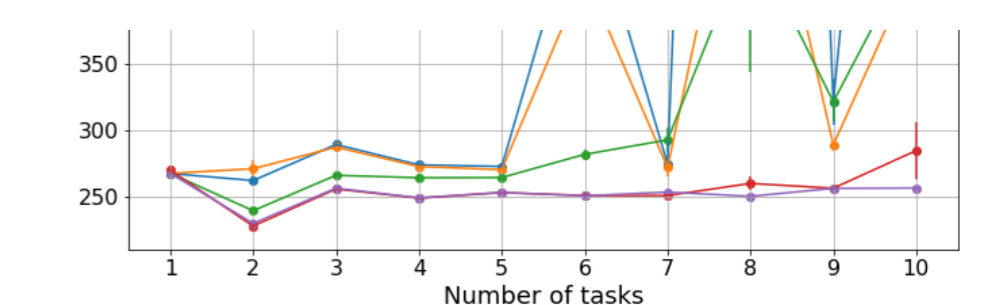
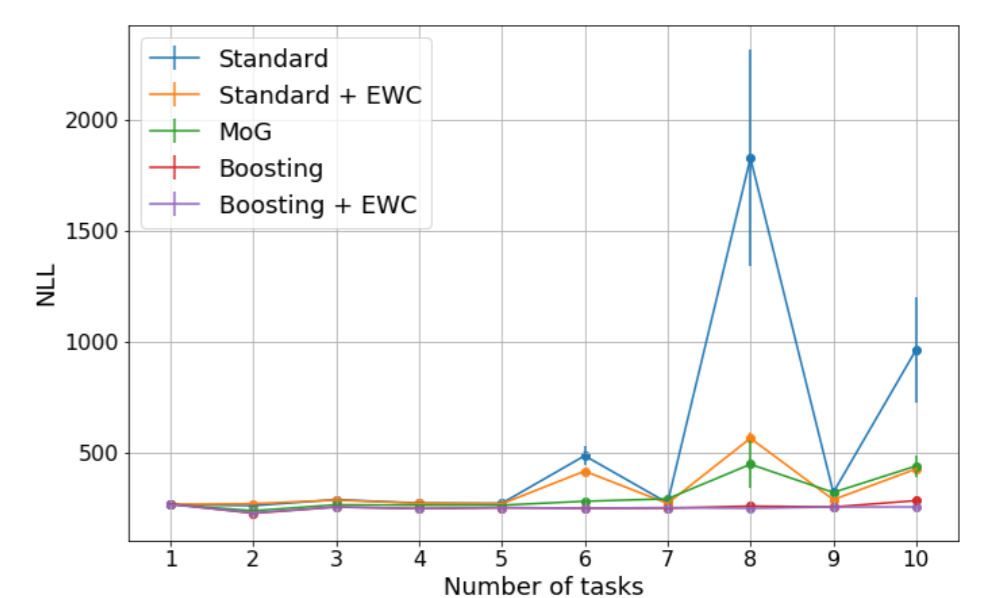
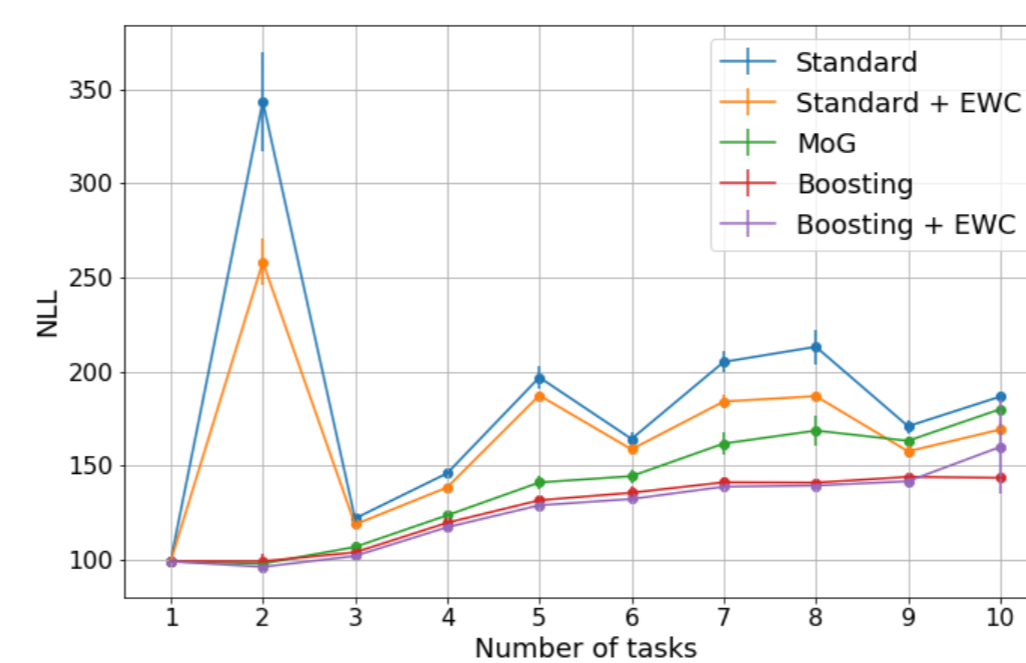
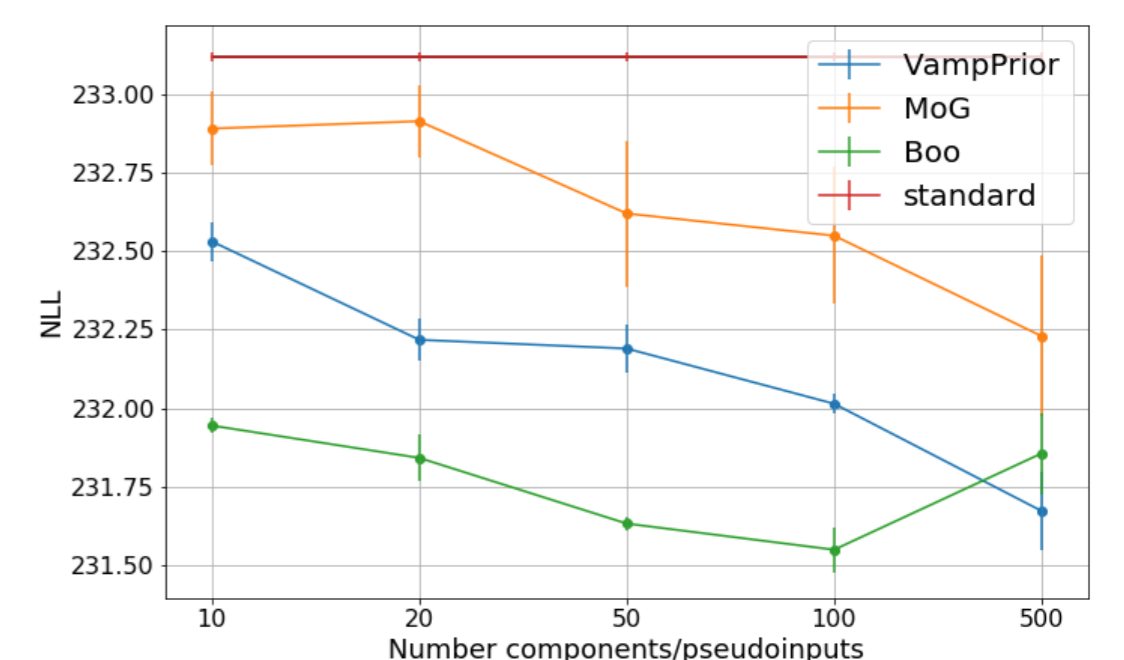
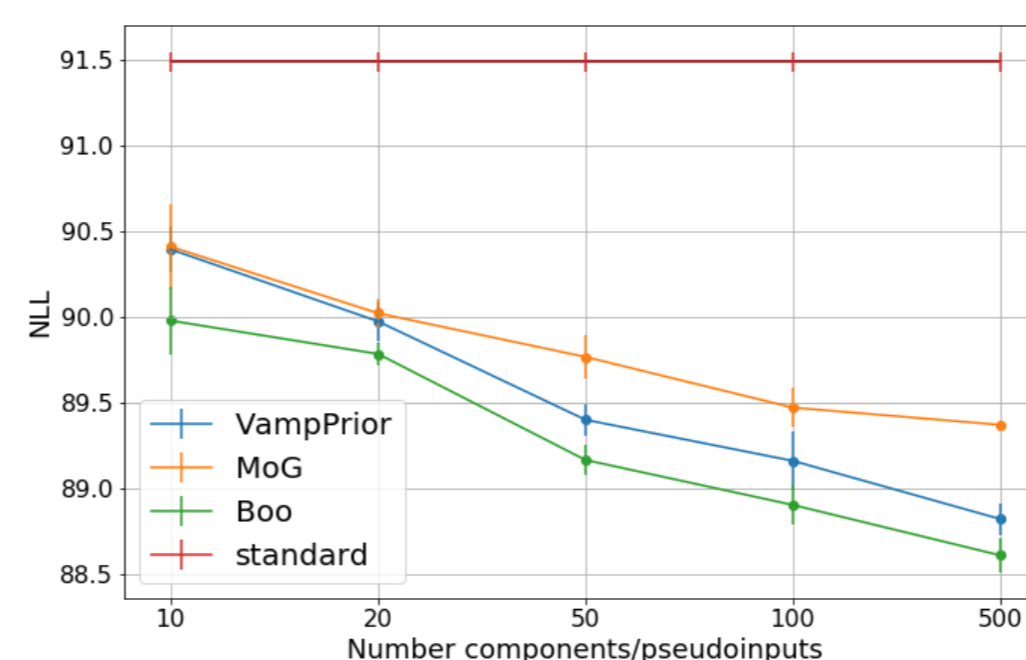
# Tasks	MNIST		Fashion MNIST	
	EWC	Boo	EWC	Boo
2	256.55	<b>100.11</b>	271.14	<b>227.83</b>
5	192.84	<b>132.08</b>	270.44	<b>253.12</b>
8	189.06	<b>140.80</b>	565.81	<b>260.05</b>
10	170.26	<b>142.92</b>	427.83	<b>284.86</b>

Table: NLL, Incremental setting

## MNIST

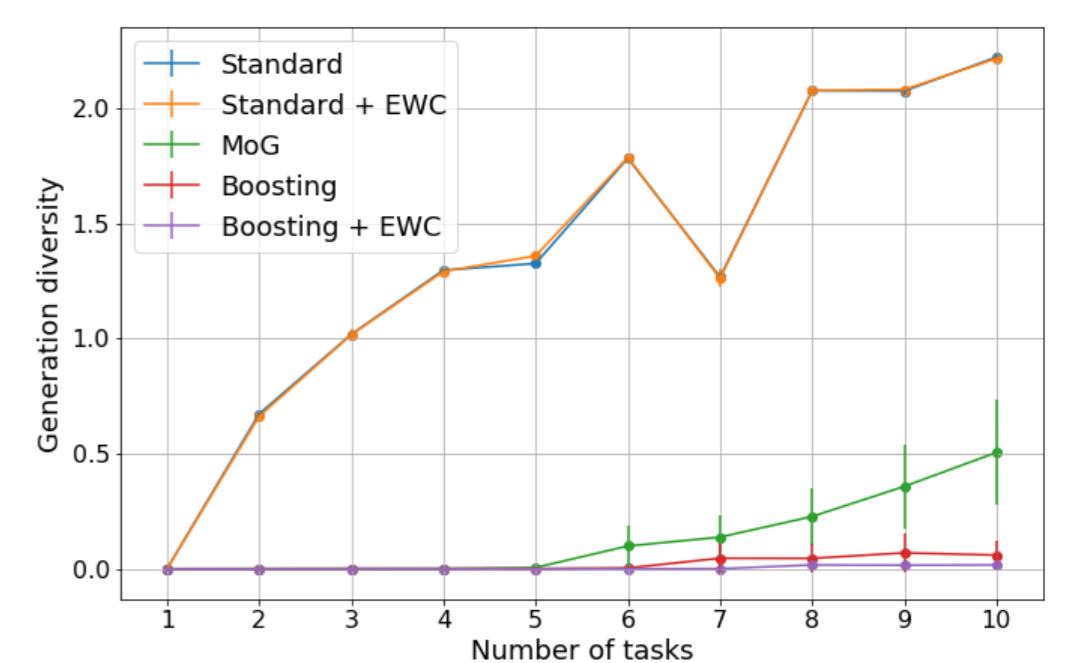
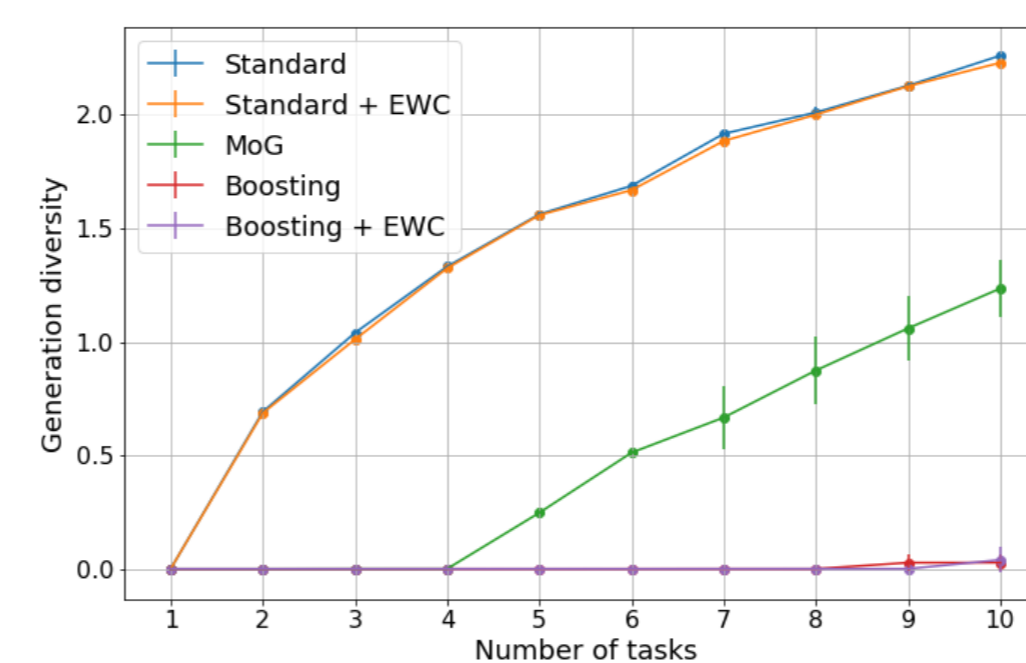
## Fashion MNIST

### IWAE bound on NLL



### Generation diversity

$$\sum_k D_{\text{KL}}(u || \bar{x}_k), u \sim \text{Be}\left(\frac{1}{K}\right), \bar{x}_k \sim \text{Be}\left(\frac{N_k}{N}\right)$$



### Generation after seeing 10 tasks (EWC and Boo)

